Frequency and Power Spectrum

The signal is fully described by its time course. In signal theory, we talk about the representation of a signal in the **time domain** (although the meaning of "time" can be e.g. distance). In a number of tasks, however, it is much more advantageous to represent the signal in the ``***frequency domain*.** The essence of the representation of the signal in the frequency domain is the expression of the signal understood as a function of time as the sum of a series of appropriately selected periodic functions. In practice, the use of trigonometric functions, i.e. sine and cosine functions, has paid off the most.

Fourier transform

Assume that *x(t)* is a signal continuous with the fundamental frequency *f*. For further considerations, it is convenient to use the circular frequency *ω=2πf* (vand by analogy with rotary motion). Such a signal can then be decomposed into the sum of phase-shifted sine waves with the frequencies *f*, *2f*, *3f*, etc. Written mathematically, the signal *x(t)* and with the fundamental frequency *ω* can be written as follows:

$$x(t) = \sum_{n=1}^{+\infty} A_n \sin(n\omega t + \varphi_n)$$

Note that the equality does not hold in general. The sum on the right side of the equation is referred to as the **Fourier series*.*** The mathematical procedure used to find the coefficients of A_n and φ_n is called ***Fourier transform***''. The sines-only form is not very convenient for further calculations, fortunately, the well-known summation formula can be used:

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

Thus, the Fourier series can also be written in the following way:

$$x(t) = \sum_{n=1}^{+\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

Where the coefficients of the Fourier series (that is, the result of the Fourier transformation) are considered to be the series of pairs (a_n, b_n). From the not very complex mathematical theory behind the Fourier transform, it follows that it is sometimes advantageous to write the Fourier series as a series of complex numbers z_n, for which the following relationship applies:

$$z_n = a_n + jb_n = A_n e^{-j\varphi_n}$$

Let's note that the complex unit can be marked both "i" and "j", the first marking is used by mathematicians and physicists, the second by technicians.

Significance in the study of linear systems

Decomposition of the signal into harmonic components is of great importance for linear systems. Linear systems are characterized by the fact that the response to the sum of two signals is actually the sum of the response to these individual signals. The response to a harmonic signal is relatively easy to investigate mathematically.

Therefore, if we are interested in the response of a linear system to an arbitrary signal, it is enough to examine the response step by step by individual components of the Fourier series and add up the result. One consequence is that no frequencies other than those contained in the signal x(t).

Frequency spectrum

The frequency spectrum of a signal is nothing but a series of ``z_n *elements described above. Because these are complex numbers, representing the sequence in a plane is not easy. Amplitude and phase spectrum are usually used. Information about the signal is carried by both spectra together, one without the other does not contain full information.*

Amplitude spectrum

The amplitude spectrum is a sequence amplitude A_n, for which the following relation with coefficients holds a_n and b_n:

$$A_n = \sqrt{a_n^2 + b_n^2}$$

Phase spectrum

The phase spectrum is a sequence of amplitudes φ_n for which the following relationship applies with the coefficients a_n and b_n *sub>*:

$$\varphi_n = \operatorname{arctg} \frac{b_n}{a_n}$$

Signal power spectrum

The power spectrum (*Power Spectrum* in English-written literature and in some areas an already domesticated Anglicism) is based on the historical electrotechnical roots of signal analysis. It is actually an answer to the question of what (thermal) power should be given by the frequency component of the signal understood as the electric voltage *u(t)* on the unit resistor. The answer is very easy, it applies to instantaneous signal performance:

$$p(t) = \frac{u^2}{R}$$

Since ``R=1 *is assumed, this symbol can be omitted. For the power carried by the nth component, the following will apply:*

$$p_n(t) = u_n^2(t)$$

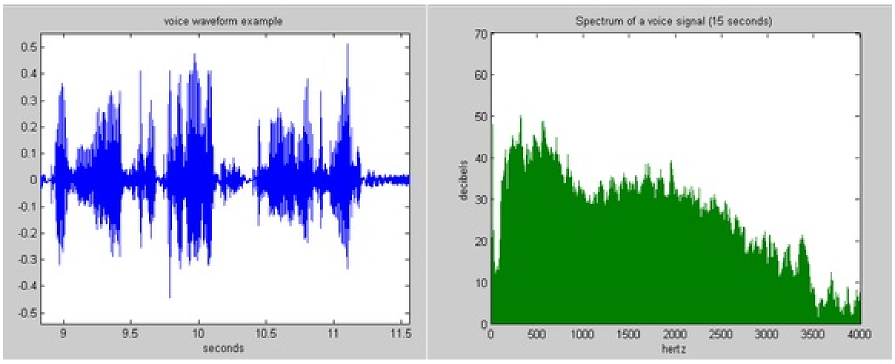
But the value $u_n(t)$ is already harmonic, since:

$$u_n(t) = A_n \sin(n\omega t + \varphi_n))$$

it can be easily shown that the (average) power carried by the nth component will have the form:

$$P_n = A_n^2$$

It differs mathematically from a simple amplitude spectrum in that it is actually the square of the amplitude spectrum. In fact, it is a significant difference, the power spectrum informs about energy ratios. An example of a biosignal in the time and frequency domain is shown in the following figure. In the left half is the time course of the sample human voice in the time domain, the right half is the same sample in the frequency domain:



A human voice sample in the time and frequency domain

Links

References

- HEŘMAN, Petr. *Biosignály z pohledu biofyziky.* 1. edition. Praha : Dúlos, 2006. 63 pp. ISBN 80-902899-7-5.
- Wikipedia. *Fourierova transformace* [online]. [cit. 2014-1-6]. <https://cs.wikipedia.org/wiki/Fourierova_transformace>.
- Wikipedia. *Frekvenční spektrum* [online]. [cit. 2014-1-6]. <https://cs.wikipedia.org/wiki/Frekven%C4%8Dn%C3%AD_spektrum>.